Quantum Computation with Trapped Ions and the "Heating Problem"

Daniel F. V. James

Theoretical Division, Los Alamos National Laboratory,
Los Alamos, NM 87545, USA

ABSTRACT

We present a brief review of the current state of the art of quantum computation with trapped ions, with particular emphasis on the problems caused by "heating" of the ions' motional degrees of freedom.

Keywords: Photonic Quantum Computing, Trapped Ions, Quantum Information

1. INTRODUCTION

Of all the technologies that have been proposed for quantum computers, argueably one of the most compelling and certainly one of the most popular is trapped ions. This scheme, discovered by Ignacio Cirac and Peter Zoller,[1] and demonstrated experimentally shortly afterwards by Monroe et al.,[2] is currently being persued by about a half-dozen groups world-wide[3] (for an overview of this work, see, for example refs.[4–6]).

A vital ingredient of trapped ion quantum computing is the ability to cool trapped ions down to their quantum ground state by sideband cooling. Using controlled laser pulses, the quantum state of the ion's collective oscillation modes (i.e. the ions *external* degrees of freedom) can then be altered conditionnally on the *internal* quantum state of the ions' valence electrons, and vice-versa. This allows quantum logic gates to performed. The current state-of-the-art (as of spring, 2000) is that two groups have suceeded in cooling strings of a few ions to the quantum ground state,[7–10] and that entanglement of up to four ions has been achieved experimentally.[11]

The accuracy of the quantum logic gates performed in trapped-ion quantum computers using Cirac and Zoller's paradigm relies cruitially on the quantum state of the ions' collective oscillatory motion. The ions must be in the

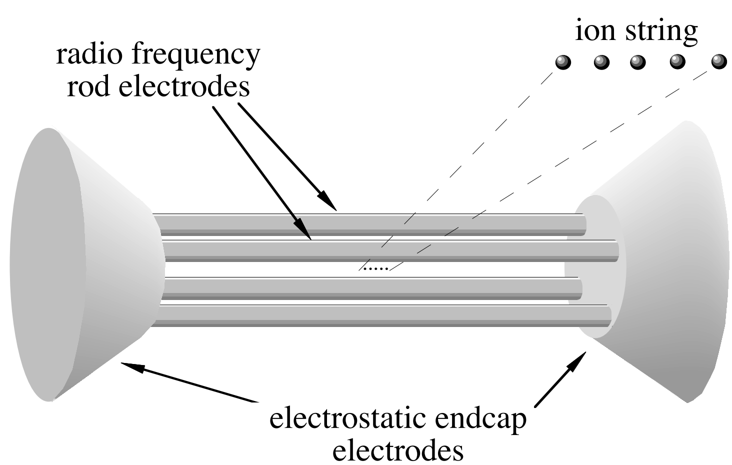

Figure 1. Schematic picture of an ion trap quantum computer. The ions are confined by a radio frequency potential established by four rod electrodes, and by an electrostatic potential due to a pair of end-cap electrodes. The ions, which are cooled by lasers to their quantum ground states, can be used to store information in their internal quantum levels. Conditional logic operations involving two or more ions can be performed by selectively exciting a quanta of the ions' collective oscillations. Control of these operations is carried out using pulses of laser light.

quantum ground states of these degrees of freedom (the quanta of which are widely referred to as phonons). If the purity of this quantum state were to be degraded by the action of external perturbations (which, given the fact that ions couple to any externally applied electric field, seems quite likely) then the accuracy of quantum operations will suffer. The maintainance of the cold ions in their oscillatory quantum ground state seems at the moment to be the biggest single problem standing in the way of advancing this field. The solution is being tackled in two ways: firstly the understanding and nuffication of the experimental causes of the "heating" of the trapped ions, and secondly the investigation of alternative schemes for performing quantum logic operations which relax the strict condition of being in the quantum ground state of the phonon modes. Here we will give a brief account of the former and describe one of the most promising schemes for the latter.

2. THE PROBLEM: HEATING DUE TO STRAY E/M FIELDS

The effect of flucuating e/m fields on trapped ions has been analyzed by various authors[12–16]; we will give a brief reprise of it here. N ions are confined in a trap. The trap is assumed to be anistropic, so that the ions lie crystalized along the axis of weak confinement (the x-axis, say). The ions' motion will be strongly coupled by the Coulomb force. Their small amplitude oscillation are described by *normal modes*, each of which is an independent harmonic oscillator.[17] Neglecting motion transverse to the x-axis, there will be a total of N such modes. We shall number these modes in order of increasing resonance frequency, the lowest ($p = 1$) mode being the center of mass mode, in which the ions oscillate as if rigidly clamped together. In the quantum mechanical description, each mode is characterized by creation and annihilation operators $\hat{a}_p^\dagger$ and $\hat{a}_p$ (where $p = 1, \ldots N$). The ions are interacting with an extrnal electric field $\mathbf{E}(\mathbf{r}, t)$. The Hamiltonian in this case is given by the expression

$$\hat{H} = i\hbar \sum_{p=1}^{N} \left[u_p(t)\hat{a}_p^\dagger - u_p^*(t)\hat{a}_p \right], \tag{1}$$

where

$$u_p(t) = \frac{ie}{\sqrt{2M\hbar\omega_p}} \sum_{n=1}^{N} E_x(\mathbf{r}_n, t) b_n^{(p)} \exp\left(i\omega_p t\right). \tag{2}$$

In eq.(2), $b_n^{(p)}$ is the n-th element of the p-th normalized eigenvector of the ion coupling matrix,[17] ω_p being its resonance frequency, and E_x is the component of the electric field along the weak axis of the trap. In what follows, the center of mass phonon mode ($p = 1$), whose frequency is equal to the frequency ω_x of the Harmonic trapping potential, will have special importance.

The frequencies at which the externally field $\mathbf{E}(\mathbf{r}, t)$ are resonant with the ions' motion is at most a few MegaHertz; the wavelengths of such radiation will therefore not be less than 100 meters or so. The separation of the ions is of the order of 10 μm, or 10^7 wavelengths. Thus spatial frequencies in the applied field on the spatial scale of the ions' separation will be very evanescent, and to a very good approximation one can assume $E_x(\mathbf{r}_n, t) \approx E_x(t)$, i.e. the field is constant over the extent of the ion string. Using the fact that $\sum_{n=1}^{N} b_n^{(p)} = \delta_{p,1}$, the interaction Hamiltonian becomes

$$\hat{H} = i\hbar u_1(t)\hat{a}_1^\dagger + h.a., \tag{3}$$

where

$$u_1(t) = \frac{ie\sqrt{N}}{\sqrt{2M\hbar\omega_x}} E_x(t) \exp\left(i\omega_x t\right), \tag{4}$$

where $\omega_x \equiv \omega_1$ is the trapping frequency along the x-axis. In other words, spatially uniform fields will only interact with the center-of-mass mode of the ions, which is physical inituitive since some form of differential force must be applied to excite modes in which ions move relative to one another.

The dynamics governed by this Hamiltonian can be solved exactly.[16] The heating rate is given by the formula

$$\begin{aligned} \frac{d\bar{n}}{dt} &= \frac{\pi N e^2 E_{RMS}^2}{M\hbar\omega_x} \mathcal{S}\left(\omega_x\right) \\ &\approx \frac{\pi N e^2 E_{RMS}^2 T}{M\hbar\omega_x} \text{ (white noise limit)} \end{aligned} \tag{5}$$

where E_{RMS} is the root mean square value of $E_x(t)$, $\mathcal{S}(\omega)$ is the power spectrum of $E_x(t)$ normalized so that $\int_0^\infty \mathcal{S}(\omega)\, d\omega = 1$, and T is its coherence time (in the white noise limit).

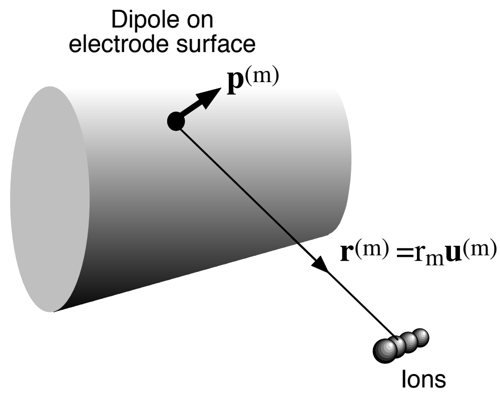

Figure 2. Schematic picture of a patch potential on the surface of a rod electrode.

2.1. Simple Model for "patch potentials"

The current conventional wisdom for the source of the electric fields causing this heating is "patch potentials" on the surface of the trap electrodes. These are regions of the electrode surface in which the electric potential varies from the average value. Little seems to be known about the physics of patch potentials fluctuating at radio frequencies. In order to gain some idea of how the trap geometry influences the heating rate, we will adopt the following simple model. The patches are isolated point dipoles situated on the surface of the electrodes (see Fig.2). The electric field generated by these dipoles *if they were in free space* is[19]

$$\mathbf{E}(t)=\sum_m \frac{1}{4\pi\varepsilon_0 r_m^3}\left[3\mathbf{u}^{(m)}\left(\mathbf{u}^{(m)}\cdot\mathbf{p}^{(m)}\right)-\mathbf{p}^{(m)}\right] \tag{6}$$

where $\mathbf{p}^{(m)}$ is the dipole moment of the m-th dipole, $\mathbf{u}^{(m)}$ is the unit vector along the line joining the m-th dipole to the center of the ion string, and r_m is the distance from the dipole to the ions. We have assumed the near-zone expression for the field, justified in these circumstances because of the long wavelengths associated with the radio frequencies present, already mentioned. We have also, for simiplicity, ignored the fact that the dipoles are on the surface of a conductor rather than in free space.

If we assume uncorrelated statistically stationary random dipoles, i.e.

$$\langle p_i^{(m)}(t)p_j^{(n)}(t+\tau)\rangle=\delta_{n,m}\delta_{i,j}\Gamma_p(\tau), \tag{7}$$

(where i and j represent cartesian coordinates, and $\delta_{i,j}$ is the Kronecker delta symbol,) then the correlation function for the random electric field along the trap axis is given by

$$\Gamma_E(\tau)=\sum_m \frac{1}{(4\pi\varepsilon_0)^2}\frac{3\mathbf{u}^{(m)}\cdot\mathbf{u}^{(m)}+1}{r_m^6}\Gamma_p(\tau). \tag{8}$$

We can assume a continuous, uniform distribution of these dipoles over the surfaces of each of the four rods, with density σ dipoles per unit area, then we can convert this sum into an integral:

$$\begin{aligned}\sum_m \frac{3\mathbf{u}^{(m)}\cdot\mathbf{u}^{(m)}+1}{r_m^6} \quad &\rightarrow \quad 8\pi a\sigma\int_{-\infty}^{\infty}\frac{1}{r(\ell)^6}\left(3\sin^2\theta(\ell)+1\right)d\ell \\ &= \quad \frac{8\pi a\sigma}{d^5}\int_{-\pi/2}^{\pi/2}\cos^4\theta\left(3\sin^2\theta+1\right)d\theta \\ &= \quad \frac{9\pi^2 a\sigma}{2d^5}. \end{aligned} \tag{9}$$

In eq.(9) d is the closest distance between the rod and the ions, a is the rod radius (we have assumed that $d\gg a$), θ is the angle as shown in fig.3 and $\ell=d\tan\theta$ is the length along the rod electrode measured from the point nearest to the ions.

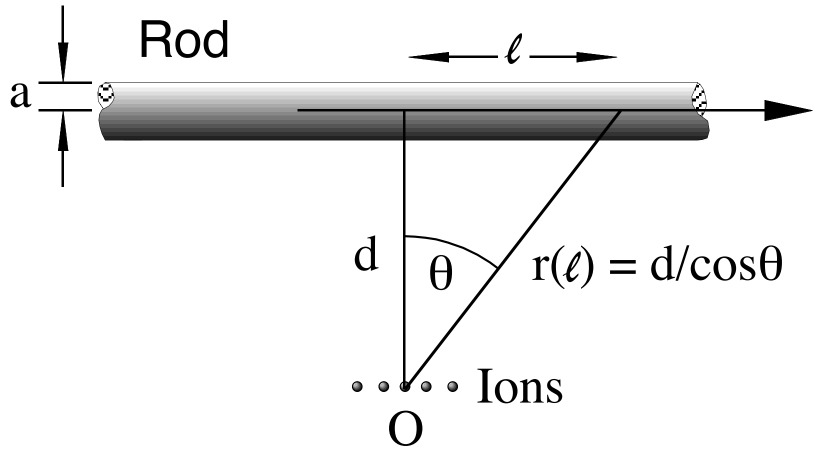

Figure 3. Diagram showing the meaning of the symbols introduced in eq.(9). The origin O is at the center of the ion string, whose length is assumed to be much less than the distance to the rod, d.

Thus, using eq.(5) we obtain the following expression for the heating rate of the ions

$$\frac{d\bar{n}}{dt} = \frac{9\pi N e^2 \sigma a P_{RMS}^2}{32 M \hbar \omega_x \varepsilon_0^2 d^5} \mathcal{S}\left(\omega_x\right). \tag{10}$$

To recap: in eq.(10) N is the number of ions, σ is the surface desnity of dipoles on the rod electrodes, a is the radius of the rod electrodes, P_{RMS} is the root mean square dipole moment of the patches, M is the mass of the ions, ω_x is the angular trapping frequency in the axial direction and d is the shortest distance between the ions and the rods.

This result is in qualitative agreement with the conslusions of Turchette *et al.*,[15] which gives both theoretical and experimental evidence that the heating rate is proportional to a high inverse power of the trap dimensions. Those authors conclude that $d\bar{n}/dt \propto d^{-4}$ rather than $d\bar{n}/dt \propto d^{-5}$ found here; however the approximations used to derive our results are so crude that no particular significance should be attached to the difference.

2.2. Quantum Computing using "higher" phonon modes

The analysis of the heating of ions presented in the previous section is directly linked a conceptually simple method for quantum computing with trapped ions in a manner which avoids the heating problem.[7,16] The "higher" ($p > 1$) modes of the ions' collective oscillations can be utilized in place of the center of mass ($p = 1$) mode originally considered by Cirac and Zoller. The pulse sequence required is exactly that proposed by those authors, with the slight added complication that different laser frequencies (i.e. the sideband corresponding the stretch mode in question) must be employed, and that the laser-ion coupling varies between different ions for the higher modes,[17] requiring different pulse durations for different ions.

Experimentally the "higher" modes of the two-ion system are observed to have heating times in excess of 5 μsec, as opposed to heating times of less than 0.1 μsec for the center of mass modes,[7] confirming that they are indeed well isolated from the influence of external heating fields, and can be used as a reliable quantum information bus.

The heating of the center of mass mode has an important indirect influence. As this mode becomes more and more excited, the wavefunction of the ions becomes more spatially smeared-out, causing a random phase shift of the ions. This effect is analogous to the Debye-Waller effect in X-Ray crystallography.[7] One possible solution for this problem has been proposed,[18] namely the use of *sympathetic cooling* by a separate species of ion, allowing the excitation of the center of mass mode to be reduced and kept constant. This scheme however poses the problem of devising a method of loading a trap with an ion of a distinct species and providing a second set of lasers to cool it.

3. THE PARADIGM SHIFT OF SORENSEN AND MOLMER: A CURE FOR HEATING?

Mølmer and Sørensen have proposed related techniques for creating both multi-ion entangled states[20] and for quantum computation[21,22] with ions in thermal motion. It relies on the *virtual* excitation of phonon states, in a manner analgous to the virtual excitation of some exited state of an atom or molecule in Raman processes. Laser fields with two spectral components detuned equally to the red and to the blue of the atomic resonance frequency are applied to a pair of ions in the trap. The interaction is described by the following Hamiltonian

$$\begin{aligned}\hat{H}_I &= \hbar\Omega\hat{J}^{(+)}\left\{1+i\eta\left(\hat{a}_1 e^{-i\omega_x t}+\hat{a}_1^\dagger e^{i\omega_x t}\right)\right\}\cos(\delta t)+h.a. \\ &= \hbar\Omega e^{i\delta t}\hat{J}_x - \hbar\Omega\eta e^{i(\delta+\omega_x)t}\hat{a}^\dagger\hat{J}_y - \hbar\Omega\eta e^{i(\delta-\omega_x)t}\hat{J}_y\hat{a}+h.a.\end{aligned} \tag{11}$$

In this equation δ is the detuning of the laser beam from the resonance frequnecy of the two level system, η is the Lamb-Dicke coefficient (which is a dimensionless quantity that charaterises the coupling between the laser and the ions' motion) and Ω is the Rabi frequency (proportional to the laser field strength). The operator $\hat{J}^{(+)}=\sum_{Ions}\hat{\sigma}^{(+)}$ is the collective raising operator for the two or more ions involved in the gate, and , as usual $\hat{J}_x=(\hat{J}^{(+)}+\hat{J}^{(+)\dagger})/2$ and $\hat{J}_y=(\hat{J}^{(+)}-\hat{J}^{(+)\dagger})/2i$.

For large values of δ it is convenient to consider this interaction in terms of an effective Hamiltonian (see appendix), which neglects the effects of very rapidly varying terms. In this case, the effective Hamiltonian is

$$\begin{aligned}\hat{H}_{eff} &= \frac{\hbar\Omega^2\eta^2}{(\delta+\omega_x)}\left[\hat{J}_y\hat{a},\hat{a}^\dagger\hat{J}_y\right]+\frac{\hbar\Omega^2\eta^2}{(\delta-\omega_x)}\left[\hat{a}^\dagger\hat{J}_y,\hat{J}_y\hat{a}\right] \\ &= \frac{\hbar\Omega^2\eta^2}{(\delta-\omega_x)}\left(\frac{2\omega_x}{\delta+\omega_x}\right)\hat{J}_y^2.\end{aligned} \tag{12}$$

This interaction is equivalent to a conditional quantum logic gate preformed between the two ions, and can be used to create multiparticle entangled states.

This scheme is very attractive because, while it has the possibility of being scalable to many ions, its operation is *independent* of the occupation number of the phonon modes, and so its fidelity is not degraded by excitation *during* the gate operations themselves. Its chief drawback seems to be the time taken to perform gate operations. For example,[21] the population oscillations during a noise-resilient entangling operation have an effective Rabi frequency of approximately $4500\omega_x$. Given that trap frequencies must be of the order of $\omega_x \sim (2\pi)500$ kHz in order for the ions to be individually resolvable by focused lasers * , this implies a gate time of the order of 50 milliseconds. As explained in ref.,[22] it is possible to decrease this time by reducing the detuning δ of the laser, at the cost of increasing the susceptibility of this scheme to heating during gate operations. Nevertheless Mølmer and Sørensen's scheme is a very compelling idea, and has been used experimentally to create entangled states of four ions.[11]

4. ASSESMENT OF THE TECHNOLOGY

The various schemes for quantum computation with trapped ions in principle meet many of the criteria for scalable quantum computation technology. Here we discuss the various criteria one by one.

4.1. Initialization

The quantum information register (the ions) and the quantum information bus (the phonon modes) can be initialized reliably using laser cooling and optical pumping. Important aspects of these techniques have already been demonstrated experimentally. The "hot" ion scheme discussed here (as well as the other proposed hot ion quantum computing schemes[23,24]) may ease the stringent requirements on preparation of the initial state of the ions collective oscillation modes.

4.2. Gate Operations

Quantum logic can be performed using laser control of the quantum states of the ions internal and external degrees of freedom, requiring pulses of known duration and strength focused acurately on individual ions. Methods for aleiviating the laser focusing problem by altering the ions resonance frequency by various means such as non-uniform electric or magnetic fields have been proposed.[6,25] The ability to address individual ions with laser beams and control their quantum states has been demonstrated experimentally by two groups using various means.[8,9]

*It is not necessary to resolve ions individually for this scheme to be used to create entanglement; however some form of differential laser addressing will be necessary in order to perform quantum computations involving more than two qubits.

4.3. Isolation from the Environment

The internal degrees of freedom of the ions, in which the quantum information is stored, have very long decoherence times (especially when Raman transitions form the basis of the single-qubit operations.) The principal form of enviromental disrupion suffered by ion traps is disturbance of the motional degrees of freedom.

The "higher modes" scheme is well isolated from the environment, except for the indirect influence of the Debye-Waller effect. The Mølmer-Sørensen scheme is not intrinsically isolated from the environment, but avoid its influence by avoiding significant excitation of the phonon states.

4.4. Error correction

There is nothing instrinsic that will rule out implimentation of fault tolerant quantum computation in ion traps when sufficient numbers of ions become availible. Ancilla ions can be prepared in their quantum ground state independent of other ions in the register. The use of multiple stretch modes (there are N-1 such modes in the weak trapping direction), allows quantum gates to be performed in parallel. Read out can be performed at intermediate stages during calculations without destroying the qubit being read, or disturbing other ions in the register unduly (there will be recoil during the read out that has the possibility of excitation of the oscillatory modes).

4.5. Read Out

The read-out of the quantum state of ions using a cycling transition has been demonstrated experimentally with high efficiency and reliability.[2] Indeed these experiments are the *only* ones in which high efficiency strong measurement of a single quantum system (as opposed to an ensemble of systems) has been performed.

4.6. Scalability

The ultimate number of ions that can be stored in a string in an ion trap and used for quantum computation is limited by a number of factors. Probably the most important is growing complexity of the sideband spectrum as the number of ions grows. Even in the case of highly anisotropic traps (in which transverse oscillations can be neglected) the number of oscillation modes is equal to the number of ions, and each mode has a distinct frequency, with an infinite ladder of excitation resonances. In addition one has to take into account multi-phonon resonances; the whole leading very complicated structure in frequency space. The extent to which this "spectrum of death"[26] can be understood and exploited, by systematic identication of resonances, careful bookeeping and tayloring of Lamb-Dicke coefficients remains to be seen. Other effects which place an upper bound on the number of ions in a single register is that fact that the spatial separation of the ions decreases $\propto N^{-0.56}$ making their spatial resolution by a focused laser beam more and more difficult.[27] Another, more definite upper bound on the number of ions that can be stored in a linear configuration is the onset of a phase transition to a more complex configuration such as a zig-zag[28]; for traps optimized for quantum computation with singly ionized calcium this occurs at about 170 ions. It is however argueable whether or not the onset of instabilities makes quantum computation impossible.

If only small numbers of ions can be reliably used for quantum computation in a single ion trap, multiple traps will be needed for large-scale devices. DeVoe[29] has proposed fabricating multiple elliptical traps, each suitable for a few dozen ions, on a substrate with a density of 100 traps/cm^2. Some form of reliable, high efficiency quantum communication channel to link the multiple traps would need to be implemented.[30–32] An alternative scheme has been proposed by Wineland *et al.*[6,33] in which two traps are used. One trap is used to store a large number of ions in a readily accessible manner (e.g. in an easily rotated ring configuration); each of these ions form a qubit of the register of the quantum computer. When a gate operation is to be performed, the two involved ions are extracted from the storage trap by applying static electric fields in an appropriate controlled manner, and transfered to a separate logic trap where they can be cooled and quantum logic operations can be performed on them. The cooling could be done sympathetically by a third ion of a separate species stored in the logic trap (thereby preserving the quantum information stored in the two logic ions which otherwise would be lost during cooling); in these circumstances either the original Cirac-Zoller scheme or any of the "hot gates" schemes described here can be used as the mechanism for peforming the logic.

In conclusion, the variety and richness of the quantum computing schemes that have been devised for ion traps illustrates the great flexibility of this technology. Uniquely amoungst the proposals for quantum computing technology, the question for ion traps is not "does it work?" but rather "how far can it go?"

APPENDIX A: EFFECTIVE HAMILTONIANS FOR DETUNED SYSTEMS

We start with the Schrödinger equation in the interaction picture, i.e.

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}_I(t)|\psi(t)\rangle \tag{13}$$

The *formal* solution of this first order partial differential equation is

$$|\psi(t)\rangle = |\psi(0)\rangle + \frac{1}{i\hbar}\int_0^t \hat{H}_I(t')|\psi(t')\rangle dt'. \tag{14}$$

Subtituting this result back into eq.(13), we obtain

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}_I(t)|\psi(0)\rangle + \frac{1}{i\hbar}\int_0^t \hat{H}_I(t)\hat{H}_I(t')|\psi(t')\rangle dt' \tag{15}$$

If we assume that the interaction Hamiltonian is strongly detuned, in the sense that $\hat{H}_I(t)$ consists of a number of highly oscillative terms, then to a good approximation the first term on the right hand side of eq.(15) can be neglected, and we can adopt a Markovian approximation for the second term, so that the evolution of $|\psi(t)\rangle$ is approximately governed by the following equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle \approx \hat{H}_{eff}(t)|\psi(t)\rangle, \tag{16}$$

where

$$\hat{H}_{eff}(t) = \frac{1}{i\hbar}\hat{H}_I(t)\int \hat{H}_I(t')dt', \tag{17}$$

where the indefinite integral is evaulated at time t without a constant of integration. These arguments can be placed on more rigorous footing by considering the evolution of a time-averaged wavefunction.

We will now assume that the interaction Hamilitonian consists explicitly of a combination of harmonic time varying components, i.e.

$$\hat{H}_I(t) = \sum_m \hat{h}_m \exp(i\omega_m t) + h.a., \tag{18}$$

where $h.a.$ stands for the the hermitician adjoint of the perceeding term, and the frequencies ω_m are all distinct (i.e. $m \neq n \Leftrightarrow \omega_m \neq \omega_n$). In this case the effective Hamiltonian $\hat{H}_{eff}(t)$ reduces to a simple form useful in the analysis of laser-ion interactions:

$$\begin{aligned}
\hat{H}_{eff}(t) &= \sum_{m,n}\frac{1}{i\hbar}\left(\hat{h}_m e^{i\omega_m t} + \hat{h}_m^\dagger e^{-i\omega_m t}\right)\left(\hat{h}_n\frac{e^{i\omega_n t}}{i\omega_n} + \hat{h}_n^\dagger\frac{e^{-i\omega_n t}}{-i\omega_n}\right) \\
&= \sum_{m,n}\frac{1}{-\hbar\omega_n}\left(\hat{h}_m\hat{h}_n e^{i(\omega_m+\omega_n)t} + \hat{h}_m\hat{h}_n^\dagger e^{i(\omega_m-\omega_n)t} - \hat{h}_m^\dagger\hat{h}_n e^{-i(\omega_m-\omega_n)t} - \hat{h}_m^\dagger\hat{h}_n^\dagger e^{-i(\omega_m+\omega_n)t}\right) \\
&= \sum_m \frac{1}{\hbar\omega_m}[\hat{h}_m^\dagger, \hat{h}_m] + \text{oscillating terms}.
\end{aligned} \tag{19}$$

If we confine our interest to dynamics which are time-averaged over a period much longer than the period of any of the oscillations present in the effect Hamiltonian (i.e. averaged over a time $T \gg 2\pi/\min\{|\omega_m - \omega_n|\}$) then the oscillating terms may be neglected, and we are left with the following simple formula for the effective Hamiltonian:

$$\hat{H}_{eff}(t) = \sum_m \frac{1}{\hbar\omega_m}[\hat{h}_m^\dagger, \hat{h}_m]. \tag{20}$$

ACKNOWLEDGMENTS

The author wishes to thank Gerard Milburn, Sara Schneider, Andrew White, Michael Holzscheiter, Ferdinand Schmidt-Kaler, Juergen Eschner and Dave Wineland for useful conversations and correspondence. This work was performed in part while the author was a guest at the Deptartment of Physics, University of Queensland, Brisbane, Australia. He would like to thank the faculty, staff and students for their warm hospitality. This work was funded in part by the U.S. National Security Agency.

REFERENCES

1. J. I. Cirac and P. Zoller, "Quantum Computations with Cold trapped ions", *Phys. Rev. Lett.* **74**, 4094–4097 (1995).
2. C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano and D. J. Wineland, "Demonstration of a fundamental quantum logic gate", *Phys. Rev. Lett.* **75**, 4714–4717 (1995).
3. Here is a list of groups either actively engaged on trapped-ion quantum computing that has come to the author's attention: NIST Boulder (D. J. Wineland, C. Monroe *et al.*, using Be^+); University of Innsbruck (R. Blatt *et al.* using Ca^+); Los Alamos Natl. Lab. (R. J. Hughes *et al.*, using Ca^+); IBM Almaden (R. G. DeVoe *et al.* using Ba^+); MPI Garching (H. Walther *et al.* using Mg^+); Oxford University, (A. Steane *et al.*, using Ca^+).
4. A. M. Steane, "The ion trap quantum information processor," *Applied Physics B* **64**, 623-642 (1997).
5. R. J. Hughes, D. F. V. James, J. J. Gomez, M. S. Gulley, M. H. Holzscheiter, P. G. Kwiat, S. K. Lamoreaux, C. G. Peterson, V. D. Sandberg, M. M. Schauer, C. M. Simmons, C. E. Thorburn, D. Tupa, P. Z. Wang and A. G. White, "The Los Alamos trapped ion quantum computer experiment", *Fortschritte der Physik*, **46**, 329 (1998).
6. D. J. Wineland, C. Monroe, W. M. Itano, D. Leibfried, B. King, and D. M. Meekhof, "Experimental issues in coherent quantum-state manipulation of trapped atomic ions," *J. Res. Natl. Inst. Stand. Technol.* **103**, 259 (1998).
7. B. E. King, C. S. Wood, C. J. Myatt, Q. A. Turchette, D. Leibfried, W. M. Itano, C. Monroe and D. J. Wineland, "Cooling the collective motion of trapped ions to initialize a quantum register," *Phys. Rev. Lett.* **81**, 1525–1528 (1998).
8. Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe and D. J. Wineland, "Deterministic entanglement of two trapped ions"*Phys. Rev. Lett.* **81**, 3631–3634 (1998).
9. H. C. Nagerl, D. Leibfried, H. Rohde, G. Thalhammer, J. Eschner, F. Schmidt-Kaler and R. Blatt, "Laser addressing of individual ions in a linear ion trap," *Phys. Rev. A* **60**, 145-148 (1999).
10. C. Roos, T. Zeiger, H. Rohde, H. C. Nagerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler and R. Blatt, "Quantum state engineering on an optical transition and decoherence in a Paul trap," *Phys. Rev. Lett.* **83**, 4713-4716 (1999).
11. C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland and C. Monroe, "Experimental entanglement of four particles," *Nature* **404**, 256-259 (2000).
12. S. K. Lamoreaux, "Thermalization of trapped ions: a quantum perturbation approach," *Phys. Rev. A* **56** 4970-4975 (1997).
13. S. Schneider and G. J. Milburn, "Decoherence in ion traps due to laser intensity and phase flucuations" *Phys. Rev. A* **57** 3748-3752 (1998) and "Decoherence and fidelity in ion traps with fluctuating trap parameters," *Phys. Rev. A* **59** 3766-3744 (1999).
14. M. E. Gehm, K. M. O'Hara, T. A. Savard and J. E. Thomas, "Dynamics of noise-induced heating in atom traps," *Phys. Rev. A* **58**, 3914-3921 (1999).
15. Q. A. Turchette, D. Kielpinski, B. E. King, D. Leibfried, D. M. Meekhof, C. J. Myatt, M. A. Rowe, C. A. Sackett, C. S. Wood, W. M. Itano, C. Monroe, and D. J. Wineland, "Heating of trapped ions from the quantum ground state," submitted to *Phys. Rev. A* (2000); quant-ph/0002040.
16. D. F. V. James, "Theory of Heating of the Quantum Ground State of Trapped Ions," *Phys. Rev. Lett.* **81**, 317-320 (1998).
17. D. F. V. James, "Quantum dynamics of cold trapped ions with application to quantum computation," *Appl. Phys. B* **66**, 181-190 (1998).

18. D. Kielpinski, B. E. King, C. J. Myatt, C. A. Sackett, Q. A. Turchette, W. M. Itano, C. Monroe, D. J. Wineland and W. H. Zurek, "Quantum Logic Using Sympathetically Cooled Ions," submitted to *Phys. Rev. A* (quant-ph/9909035).
19. J. D. Jackson, *Classical Electrodynamics* (3rd Edition, Wiley, New York, 1999), equation (9.20), page 411.
20. K. Mølmer and A. Sørensen, "Multiparticle entanglement of hot trapped ions," *Phys. Rev. Lett.* **82**, 1835-1838 (1999).
21. A. Sørensen and K. Mølmer, "Quantum computation with ions in thermal motion," *Phys. Rev. Lett.* **82**, 1971-1974 (1999).
22. A. Sørensen and K. Mølmer, "Entanglement and quantum computation with ions in thermal motion," submitted to *Phys. Rev. A* (2000); quant-ph/0002024.
23. J. F. Poyatos, J. I. Cirac and P. Zoller, "Quantum gates with "Hot" Trapped Ions," *Phys. Rev. Lett.* **81**, 1322-1325 (1998).
24. S. Schneider, D. F. V. James and G. J. Milburn, "Quantum controlled-NOT gate with 'hot' trapped ions," *J. Mod. Opt.*, **47** 499-506 (2000).
25. D. Leibfried, "Individual addressing and state readout of trapped ions utilizing micromotion", *Phys. Rev. A* **60**, 3335 (1999).
26. This evocative phrase was introducted by C. Monroe in public lectures to describe the complex side-band spectrum of trapped Be ions.
27. T. P. Meyrath and D. F. V. James, "Theoretical and numerical studies of the positions of cold trapped ions," *Phys. Lett. A* **240**, 37-42 (1998).
28. D. G. Enzer, M. M. Schauer, J. J. Gomez, M. S. Gulley, M. H. Holzscheiter, P. G. Kwiat, S. K. Lamoreaux, C. G. Peterson, V. D. Sandberg, D. Tupa, A. G. White, R. J. Hughes and D. F. V. James, "Observation of scaling phenomena for phase transitions in linear trapped ion crystals," preprint (2000).
29. R. G. DeVoe, "Elliptical ion traps and trap arrays for quantum computation," *Phys. Rev. A* **58**, 910-914 (1996).
30. S. J. van Enk, J. I. Cirac and P. Zoller, "Photonic channels for quantum communication," *Science* **279**, 205-208 (1998).
31. J. Zhen and G. Guang-Can, *Acta Phys. Sinica* **7**, 437 (1998)
32. A. G. White, P. G. Kwiat, and D. F. V. James, in *Proceedings of the Workshop on Mysteries, Puzzles, and Paradoxes in Quantum Mechanics*, ed. by R. Bonifacio (American Institute of Physics, New York, 1999), p. 268.
33. D. J. Wineland, C. Monroe, D. M. Meekhof, B. E. King, D. Leibfried, W. M. Itano, J. C. Bergquist, D. Berkeland, J. J. Bollinger, and J. Miller, "Entangled states of atomic ions for quantum metrology and computation," in *Atomic Physics 15*, ed. by H. B. van Linden van den Heuvell, J. T. M. Walraven and M. W. Reynolds, (Proc. 15th Int. Conf. on Atomic Physics, Amsterdam) (World Scientific, Singapore, 1997) pp. 31-46.